Correlation between non-Gaussian statistics of a scalar and its dissipation rate in turbulent flows

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(Received 21 March 2006; published 11 July 2006)

This paper reports an experimental study on the correlation between the deviation from Gaussianity of the probability density function (PDF) of a fluctuating scalar and the dependence of the scalar dissipation on the scalar itself in turbulent flows. The study demonstrates that the departure of the scalar PDF from Gaussianity reflects the degree to which the dissipation rate depends statistically on the scalar. Of important significance, present results obtained from wake and jet flows, together with those deduced from previous work on various turbulent flows, appear to point to a generic expression for the total correlation. This expression suggests that the analytical result of O'Brien and Jiang [Phys. Fluids A **3**, 3121 (1991)], derived for homogeneous turbulence, should be also valid for inhomogeneous turbulence. That is, the statistical independence of the scalar dissipation and the scalar itself appears to act as the sufficient and necessary condition for the scalar PDF to be Gaussian in any stationary turbulence. It follows that the independence assumption, often used in combustion modeling, is reasonable only in the flow region where the scalar PDF is closely Gaussian.

DOI: 10.1103/PhysRevE.74.016301 PACS number(s): 47.27.T−

I. INTRODUCTION

The advantages of the probability density function (PDF) method for statistical description of reacting turbulent flows were indicated by Hawthorne *et al.* [1] nearly 60 years ago. Since then, PDF methods have been investigated and used extensively (see, e.g., Pope [2,3], O'Brien [4] and Libby and Williams [5]). PDFs of turbulent (velocity and scalar) quantities themselves have received attention, too, with both experimental (e.g. [6–10]) and numerical (e.g. [11–13]) data reported. It was suggested that, in general, the passive scalar PDF is non-Gaussian, and not fully determined by the advecting velocity field [9–11]. This has changed the classic notion that the statistics of the passive scalar field are Gaussian and faithfully reflect the velocity field in homogeneous turbulence [14].

The experimental observation of the temperature PDF with exponential tails in Rayleigh-Benard convection [15–17] drawn attention to the joint statistics of the scalar fluctuation θ and its dissipation rate $\varepsilon_\theta[\equiv\alpha(\partial\theta/\partial x_i)(\partial\theta/\partial x_i)$, where α is the molecular diffusivity and $i=1,2,3]$ in various turbulent flows in the 1990s [6–13]. Concurrently, theorists were also stimulated to study the analytical form of the scalar PDF [18–23]. Sinai and Yakhot [18] found a relation for a passive scalar θ in homogeneous decaying turbulence without a mean gradient, expressing the limiting scalar PDF $(t\rightarrow\infty)$ in terms of $\langle\varepsilon_\theta|\theta\rangle$, the conditional expectation of ε_θ based on particular values of θ. A similar PDF form was obtained for nondecaying homogeneous turbulence with a constant mean scalar gradient [19]. For plane turbulent shear flows with a high mean gradient, Klimenko [20] applied boundary layer asymptotic analysis and obtained an approximation of the scalar PDF which is similar to the form of Sinai and Yakhot [18]. It is also similar to the form of the PDF suggested by Kuznetsov and Sabelnikov [21] for high-gradient turbulent shear flows. Pope and Ching [22] have derived an exact expression relating the PDF of any stationary random process $\theta(t)$ and the conditional expectation of $(\partial\theta/\partial t)^2$. Since $\partial\theta/\partial t$ in turbulence is correlated to the spatial $\partial\theta/\partial x_i$ through the governing equation, statistically $(\partial\theta/\partial t)^2$ is never independent of ε_θ [10]. That is, $\langle(\partial\theta/\partial t)^2|\theta\rangle$ must be correlated with $\langle\varepsilon_\theta|\theta\rangle$; hence the PDF of θ should be generally connected to $\langle\varepsilon_\theta|\theta\rangle$. In this context, it is expected that the non-Gaussian behavior of θ should be controlled largely by the statistical dependence of ε_θ and θ, irrespective of flow type.

The motivation of the present research stemmed from both the analytical work [18–23] and previous observations of the non-Gaussian scalar in various flows. There are two specific aims for this paper. The first is to examine the detailed statistical dependence of ε_θ on θ by directly measuring $\langle\varepsilon_\theta|\theta\rangle$ in two turbulent flows, i.e., a circular cylinder wake and a round jet (through their center planes). The second aim is to investigate the correlation between this dependence and the departure of the PDF from Gaussian in these flows. The passive temperature was selected to act as a scalar advected by turbulent flows.

II. EXPERIMENTAL DESCRIPTION

For generating a wake flow, a circular (brass) cylinder of diameter $D_w=12$ mm was placed in a free stream, with $U_\infty\approx4$ m/s, in a close-return, low-turbulence ($\approx0.05\%$) wind tunnel with a 3-meters-long (rectangular) working section (640 mm × 360 mm). The cylinder was installed in the midplane and spanned the full width of the working section, 200 mm from the exit plane of the contraction. As a result, there was a blockage of about 3.3% and an aspect ratio of 53.3. The cylinder was slightly heated so that temperature above ambient could be treated as a passive scalar in the near wake. The Reynolds number Re_w $(\equiv U_\infty D_w/\nu)$ for the present wake was approximately 3200.

For a jet flow, a smooth contraction nozzle of diameter $D_j=14$ mm was used; see Mi *et al.* [24] for a more detailed description of the nozzle. The original air through the vertical nozzle was warmed by a heater before entering the jet facility so that the jet initial temperature was about 40 °C above ambient. The exit Reynolds number Re_j ($\equiv U_o D_j/\nu$, where U_o is the exit bulk velocity) was about 16 000.

Present measurements were conducted between $x_1/D_w=10$ and $x_1/D_w=100$ in the wake (across which the Taylor-microscale-based Reynolds number $\mathrm{Re}_\lambda \approx 10$–70) and between $x_1/D_j=20$ and 70 in the jet ($\mathrm{Re}_\lambda \approx 20$–200). Here x_1 is the streamwise coordinate or distance from the cylinder axis (wake) or the nozzle exit (jet), whereas x_2 and x_3 below are the lateral and spanwise coordinates for the wake or radial and azimuthal coordinates for the jet.

The temperature fluctuation θ and its spatial derivatives $\partial\theta/\partial x_1$ and $\partial\theta/\partial x_2$ were simultaneously measured using a three-cold-wire probe. The Wollaston wire (Pt-10%Rh) was used, with diameter of 0.63 μm and effective length of about 0.6–0.7 mm. All the wires were aligned in the spanwise (wake) or azimuthal (jet) x_3 direction; wires 1 and 2 were separated in the lateral (wake) or radial (jet) x_2 direction by $\Delta x_2 \approx 0.8$ mm in an (x_2, x_3) plane and wire 3 was placed $\Delta x_1 \approx 0.8$ mm downstream behind wire 2. The Bachelor scale (η_B) measured on the wake centerline was about 0.3 mm at $x_1/D_w=20$ and 0.5 mm at $x_1/D_w=100$. On the jet axis, η_B was approximately 0.4 mm at $x_1/D_j=70$. Accordingly, the magnitudes of Δx_1 and Δx_2 are approximately $(1.6$–$3)\eta_B$ on the wake centerline and $(2$–$5)\eta_B$ on the jet axis, where η_B is minimum at any given value of x_1 in both flows. That is, the wire separations should be adequate, though not perfect, for obtaining the derivatives $\partial\theta/\partial x_1$ and $\partial\theta/\partial x_2$ by using the finite difference ratios $\Delta\theta/\Delta x_1$ and $\Delta\theta/\Delta x_2$ (Mi and Nathan [25]).

The wires were operated by in-house constant current circuits supplying 0.1 mA to each wire. The temperature signals from the circuits were offset, amplified, and then digitized using a multichannel, 12 bit analog/digital converter and a personal computer. The signals were low-pass filtered at a cutoff frequency f_c chosen on site by viewing the signal time-derivative spectrum (cf. Antonia *et al.* [26]) and sampled at $2f_c$. The record durations were about 50–60 s, which are sufficient to ensure the convergence of the high-order moments.

III. RESULTS AND DISCUSSION

We first examine the connection between the deviation of the PDF, $p(\theta)$, from Gaussian and the dependence of the scalar dissipation ε_θ on the scalar fluctuation θ. To do so, measurements of both $p(\theta)$ and $\langle\varepsilon_\theta|\theta\rangle$ are needed. For the latter, ideally all the three components $\langle(\partial\theta/\partial x_i)^2|\theta\rangle$ are required to be measured since

$$\langle\varepsilon_\theta|\theta\rangle = \alpha[\langle(\partial\theta/\partial x_1)^2|\theta\rangle + \langle(\partial\theta/\partial x_2)^2|\theta\rangle + \langle(\partial\theta/\partial x_3)^2|\theta\rangle].$$

The nondimensional conditional expectation $q(\theta) \equiv \langle\varepsilon_\theta|\theta\rangle/\langle\varepsilon_\theta\rangle$, where $\langle\varepsilon_\theta\rangle$ is the time-average of ε_θ (and thus independent of θ in the stationary turbulence), can be expressed by its components $q_i(\theta) \equiv \langle(\partial\theta/\partial x_i)^2|\theta\rangle/\langle(\partial\theta/\partial x_i)^2\rangle$ as follows:

$$q(\theta) = \frac{q_1(\theta) + K_{21}q_2(\theta) + K_{31}q_3(\theta)}{1 + K_{21} + K_{31}}, \tag{1}$$

where $K_{j1} = \langle(\partial\theta/\partial x_j)^2\rangle/\langle(\partial\theta/\partial x_1)^2\rangle (j=2,3)$. If ε_θ and θ are statistically independent, $\langle\varepsilon_\theta|\theta\rangle$ must be equal to $\langle\varepsilon_\theta\rangle$, so that $q(\theta)=1$ and $q_i(\theta)=1$, for all possible values of θ. For the present study, as indicated earlier, only two spatial derivative components, i.e., $\partial\theta/\partial x_1$ and $\partial\theta/\partial x_2$, were measured simultaneously via a three-cold-wire probe, from which $q_1(\theta)$ and $q_2(\theta)$ can be calculated. George and Hussein [27] demonstrated that the assumption of local axisymmetry of turbulence works well in both jet and wake flows. It is thusdeduced that $q_3(\theta) \approx q_2(\theta)$. Indeed, this approximation is valid in the far field of a round jet ($x_1/D_j=30$), see Fig. 7 of Mi *et al.* [8]. With this approximation, Eq. (1) reduces to

$$q(\theta) = \frac{q_1(\theta) + 2K_{21}q_2(\theta)}{1 + 2K_{21}}. \tag{2}$$

To investigate the correlation between $\langle\varepsilon_\theta|\theta\rangle$ and $p(\theta)$, all three separate sets of $\theta(t)$ from the three cold wires were used to calculate $p(\theta)$; and no difference was found in the results. Figures 1(a)–1(c) show distributions of $p(\theta)$, upper plots, and $q(\theta)$, lower plots, obtained on the centerline of the wake at $x_1/D_w=20$, 40, and 100, respectively. The Gaussian distribution (G) and the independence case $q_o=1$ are also indicated for comparison.

Clearly, at $x_1/D_w=20$ in the wake, $p(\theta)$ is highly asymmetric and strongly non-Gaussian over the whole range of θ. Correspondingly, $q(\theta)$ increases sharply with θ, and there does not exist any portion of θ over which $q(\theta)\approx 1$. Further downstream, $p(\theta)$ tends to be Gaussian in the neighborhood of $\theta^*=0$. While the flow evolves to the region $x_1/D_w \geq 40$, both $p(\theta)$ becomes closely Gaussian and $q(\theta)$ is near to $q_o=1$ at $|\theta^*| \leq 2.5$. That is, both approximations $p(\theta) \approx \frac{1}{\sqrt{2\pi}}\exp(-\frac{1}{2}\theta^{*2})$and $q(\theta)\approx 1$ hold over a nearly identical range of θ. This is demonstrated even better by plotting the ratio $K=p(\theta)/G$ together with $q(\theta)$ in lower parts of Figs. 1(a)–1(c). It is worth noting that, although the data of $q(\theta)$ are not accurate for $|\theta^*|>3$, as shown by error bars, due to rare large-amplitude scalar fluctuations, its trend there is sensible. The trend correctly suggests that the temperature dissipation, ε_θ, associated with "cold" ambient air (around the negative end of θ) is low whereas that with "hot" mixed air (the positive end of θ) is high. The above observations from the wake also apply for the present jet, see Fig. 2 for $x_1/D_j=40$. At the two locations of $x_2=0$ and $r_{1/2}$ (half radius of the jet based on the mean temperature), while the PDF distribution is about Gaussian at $|\theta^*| \leq 2$, the approximation $q(\theta)\approx 1$is valid over the similar range of θ.

The observations from Figs. 1 and 2 can be deduced even from previous data obtained in various turbulent shear flows. For example, also in a slightly heated round jet, approximations $q_i(\theta^*)\approx 1$ (where $i=1,2,3$) for $|\theta^*| \leq 2.5$ were observed by Mi *et al.* [8] at $x_1/D_j=30$ (their Fig. 7; $\mathrm{Re}_j=19\,000$) and by Tong and Warhaft [28] for $i=2$ on the centerline at $x_1/D_w=40$ (their Fig. 20; $\mathrm{Re}_j=18\,000$). The corresponding $p(\theta)$, as reported in Fig. 15(a) of [28], is nearly

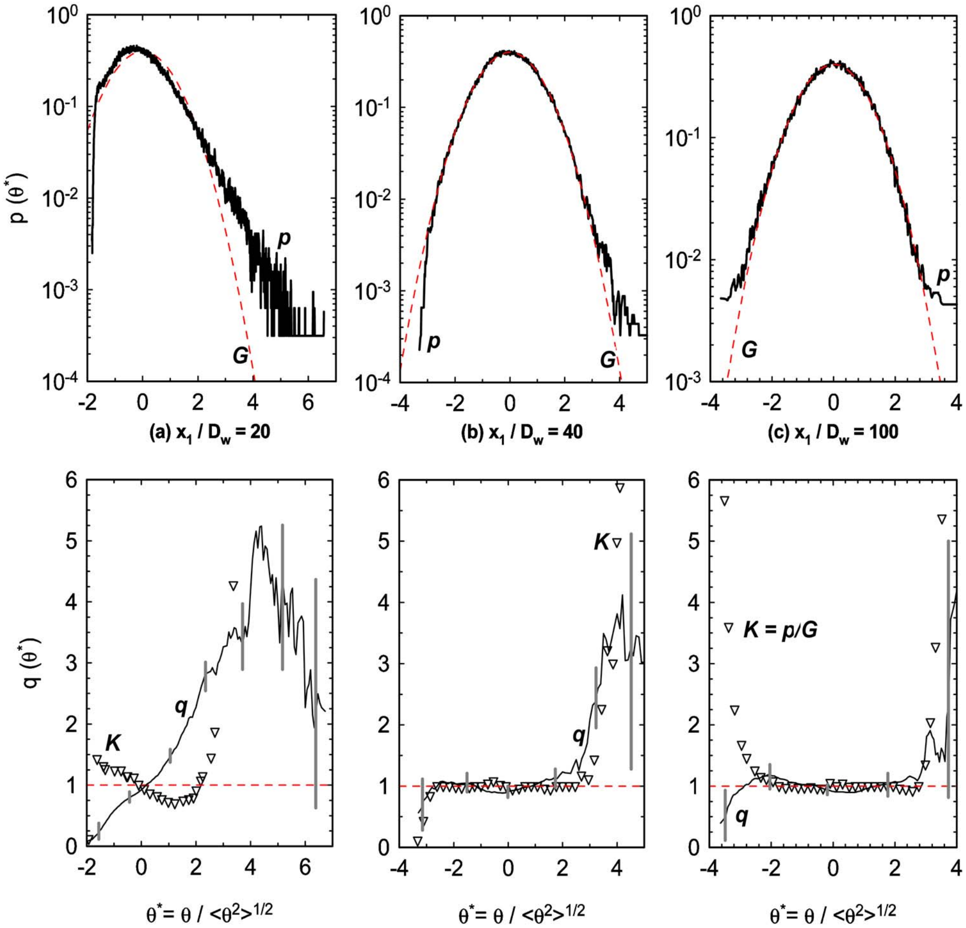

FIG. 1. (Color online) The PDF $p(\theta^*)$, upper plots, vs $q(\theta^*)$, lower plots, as a function of $\theta^* \equiv \theta/\langle\theta^2\rangle^{1/2}$ on the centerline at x_1/D_w=20, 40, and 100 in the wake flow (Re_w=3200). Symbols: - - - -, Gaussian distribution, $G=\exp(-\theta^{*2}/2)/\sqrt{2\pi}$, upper plots, and $q(\theta)=q_o=1$, lower plots; ∇, $K=p(\theta^*)/G$, lower plots; ——, (a) $p(\theta)$ and (b) $q(\theta)$; |, error bars for $q(\theta)$, lower plots.

Gaussian roughly over the same range of θ^*. The similar observation may be made from data presented in [29] for a boundary layer and a jet and in [6,7,30] for grid decaying turbulence with and without mean temperature gradients. Note, however, that the previous studies [6–8,28–30] did not investigate the correlation between$p(\theta)$ and $q(\theta)$.

It is hence deduced that, if $q(\theta)=1$ or $\langle\varepsilon_\theta|\theta\rangle=\langle\varepsilon_\theta\rangle$ over the entire range of θ, the scalar PDF should be Gaussian at all possible θ, and vice versa. Since $\langle\varepsilon_\theta|\theta\rangle=\int_0^M \varepsilon_\theta p(\varepsilon_\theta|\theta)d\varepsilon_\theta=\langle\varepsilon_\theta\rangle$=constant for all θ, where $\varepsilon_\theta\geq 0$, the conditional PDF $p(\varepsilon_\theta|\theta)$ must not be a function of θ (otherwise $\langle\varepsilon_\theta|\theta\rangle$ must depend on θ), i.e., $p(\varepsilon_\theta|\theta)\equiv p(\varepsilon_\theta,\theta)/p(\theta)=p(\varepsilon_\theta)$. That is, the equality $\langle\varepsilon_\theta|\theta\rangle=\langle\varepsilon_\theta\rangle$ for all θ means that ε_θ and θ are statistically independent. It follows that the $\varepsilon_\theta-\theta$ independence should act as a sufficient and necessary condition for the Gaussianity of $p(\theta)$ in turbu-

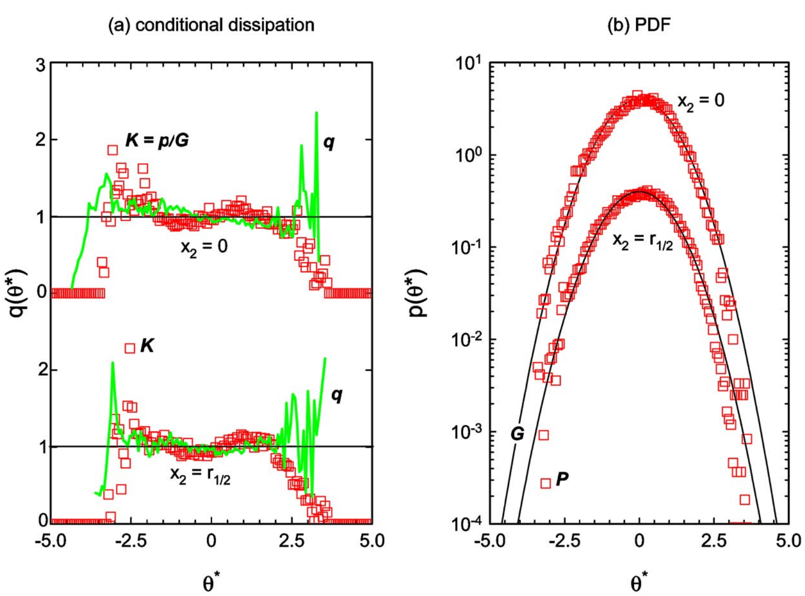

FIG. 2. (Color online) (a) The PDF $p(\theta^*)$ vs (b) $q(\theta^*)$ as a function of $\theta^*\equiv\theta/\langle\theta^2\rangle^{1/2}$ at x_2=0 and $r_{1/2}$ (half radius) in the jet flow (Re_j=16 000). Symbols: ——, (a) q_o=1 and (b) Gaussian distribution $G=\exp(-\theta^{*2}/2)/\sqrt{2\pi}$; □, (a) PDF and (b) $K=p(\theta^*)/G$; ——, (a) q.

lent jet, wake, boundary layer and grid flows. This deduction, despite no theoretical proof available, would appear to apply for any stationary turbulent flows and thus to be more general than the analytical result of O'Brien and Jiang [31] that was derived only for homogeneous turbulence.

Next we inspect the relationship between the overall departure of $p(\theta)$ from Gaussian and the extent to which ε_θ depends on θ. Two parameters are adopted here. The first one is a "correlation coefficient" between θ^2 and ε_θ defined by

$$R \equiv \frac{\langle \theta^2 \varepsilon_\theta \rangle}{\langle \theta^2 \rangle \langle \varepsilon_\theta \rangle} - 1. \tag{3}$$

This parameter, previously used in Ref. [6], differs from the classical correlation coefficient

$$\rho \equiv \frac{\langle (\theta^2 - \langle \theta^2 \rangle)(\varepsilon_\theta - \langle \varepsilon_\theta \rangle) \rangle}{\langle (\theta^2 - \langle \theta^2 \rangle)^2 \rangle^{1/2} \langle (\varepsilon_\theta - \langle \varepsilon_\theta \rangle)^2 \rangle^{1/2}}.$$

However, they have similar properties and are related by $R = (F_\theta - 1)^{1/2} (\langle \varepsilon_\theta^2 \rangle / \langle \varepsilon_\theta \rangle^2 - 1)^{1/2} \rho$. It is expected that $R \gg \rho$ as normally $F_\theta \gtrsim 3$ and $\langle \varepsilon_\theta^2 \rangle / \langle \varepsilon_\theta \rangle^2 \gg 1$. Therefore, R should be more easily estimated accurately, especially when the true ρ is close to zero. If ε_θ and θ are independent, their joint PDF $p(\varepsilon_\theta, \theta) = p(\theta) p(\varepsilon_\theta)$ and correlation $\langle \varepsilon_\theta \theta^2 \rangle = \langle \varepsilon_\theta \rangle \langle \theta^2 \rangle$, and thus both $R$and ρ must be zero. [Note that $p(\theta) = \int_{-\infty}^{\infty} p(\varepsilon_\theta, \theta) d\varepsilon_\theta$ and $p(\varepsilon_\theta) = \int_{-\infty}^{\infty} p(\varepsilon_\theta, \theta) d\theta$.] Otherwise, if ε_θ and θ are dependent, R (or ρ) $\neq 0$ even when ε_θ and θ are statistically uncorrelated or $\langle \varepsilon_\theta \theta \rangle = 0$. This can be proved here. If θ is symmetric, then $p(-\theta) = p(\theta)$ and also $p(\varepsilon_\theta, -\theta) = p(\varepsilon_\theta, \theta)$. It follows that $F(\theta) = \int_{-\infty}^{\infty} \varepsilon_\theta p(\varepsilon_\theta, \theta) d\varepsilon_\theta$ is symmetric about θ and thus that $\langle \varepsilon_\theta \theta^n \rangle$ must be zero when n is odd since $\langle \varepsilon_\theta \theta^n \rangle = \int\int \varepsilon_\theta \theta^n p(\varepsilon_\theta, \theta) d\varepsilon_\theta d\theta = \int_{-\infty}^{\infty} \theta^n F(\theta) d\theta$. For the even n, however, if ε_θ and θ are not independent, $p(\varepsilon_\theta, \theta) \neq p(\theta) p(\varepsilon_\theta)$ and $\langle \varepsilon_\theta \theta^n \rangle \neq \int\int \varepsilon_\theta \theta^n p(\varepsilon_\theta) p(\theta) d\varepsilon_\theta d\theta = \langle \varepsilon_\theta \rangle \langle \theta^n \rangle$; thus $R \equiv \langle \theta^2 \varepsilon_\theta \rangle \langle \theta^2 \rangle^{-1} \langle \varepsilon_\theta \rangle^{-1} - 1 \neq 0$. On the other hand, should θ be asymmetric, the inequality $\langle \varepsilon_\theta \theta^n \rangle \neq \langle \varepsilon_\theta \rangle \langle \theta^n \rangle$ will be valid for any n. Hence, R truly reflects the degree of the $\varepsilon_\theta - \theta$ dependence. Note that R can be estimated via its components

$$R_i \equiv \frac{\langle \theta^2 (\partial \theta / \partial x_i)^2 \rangle}{\langle \theta^2 \rangle \langle (\partial \theta / \partial x_i)^2 \rangle} - 1 \tag{4}$$

by

$$R = \frac{R_1 + K_{21} R_2 + K_{31} K_3}{1 + K_{21} + K_{31}} - 1. \tag{5}$$

The second and *new* parameter defined here is $\delta \equiv |S_\theta| + |F_\theta - 3|$, which allows the deviations of S_θ and F_θ from their Gaussian values of $S_\theta = 0$ and $F_\theta = 3$ to be combined into a single parameter. When $p(\theta)$ is Gaussian, δ must be zero. Conversely, if $\delta = 0$, then $S_\theta = 0$ and $F_\theta = 3$ so that $p(\theta)$ is expected to be Gaussian generally in turbulence, see Jayesh and Warhaft [6]. Otherwise, δ is always positive, i.e., $\delta > 0$. Obviously, the magnitude of δ is a measure of the degree to which $p(\theta)$ deviates from Gaussian.

Figure 3 reports several sets of the $\delta - R$ data obtained in both the wake and jet flows along their centerlines or across

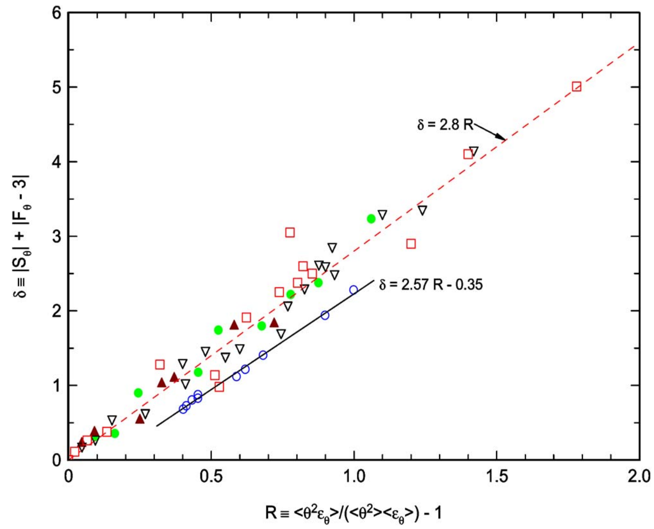

FIG. 3. (Color online) Relationship between δ and R. Plane wake ($\mathrm{Re}_w \approx 3200$, present): •, along the centerline at $x_1/D_w = 5$–100; □, across the flow at $x_1/D_w = 20$, 40, and 100. Circular jet ($\mathrm{Re}_w \approx 16\,000$, present): ▲, along the centerline at $x_1/D_j = 5$–100; ∇, across the flow at $x_1/D_j = 20$ and 40. Grid turbulence: ○, derived from Ref. [6].

them at different values of x_1 or downstream distances. (Note here that present experimental uncertainties of both δ and R were estimated, roughly with $[\delta] = \pm 1\% - \pm 3\%$ and $[R] = \pm 3\% - \pm 8\%$.) Also presented in the figure is the $\delta - R_1$ relationship which has been estimated from Jayesh and Warhaft [6] for decaying grid turbulence with a mean temperature gradient $\partial \langle \Theta \rangle / \partial x_2 = 6.06$ K/m and a velocity $U_1 = 8.9$ m/s. In that case, $\partial \theta / \partial x_1$ was obtained from $\partial \theta / \partial t$ via Taylor's hypothesis. As estimated from their PDF data reproduced, their $|S_\theta|$, not reported, should be ≤ 0.1 so that the approximation $\delta \approx |F_\theta - 3|$, where F_θ was given in Jayesh and Warhaft [6].

Figure 3 demonstrates that δ and R may be well linearly related by

$$\delta \approx 2.8R \tag{6}$$

for the present wake and jet flows. Since the data were obtained almost everywhere in the two flows, the result should represent a general feature of the scalar property in turbulence. We hence believe that relation (6) is generic, although the factor of 2.8 might be only empirical based on the present measurements. However, we also note that, for the homogeneous turbulence of Jayesh and Warhaft [6], while a linear relation is evident between δ and R_1, i.e., $\delta \approx 2.57 R_1 - 0.35$, this result is unexpected because the extrapolation to $R_1 = 0$ does not lead to the expected $\delta \approx 0$. O'Brien and Jiang [31] proved analytically that the $\varepsilon_\theta - \theta$ independence ($R = 0$) is a sufficient and necessary condition for $p(\theta)$ to be Gaussian ($\delta = 0$) for homogeneous turbulence. Since the temperature field of this flow was observed to deviate significantly from local isotropy [6], the isotropic relation $\langle \varepsilon_\theta \rangle = 3\alpha \langle (\partial \theta / \partial x_1)^2 \rangle$ is thus invalid, implying that $R \neq R_1$.This may account for the discernible difference between the present $\delta - R$ dependence and that of $\delta - R_1$ for the homogeneous turbulence.

The $\delta - R$ relation suggests that the degree to which ε_θ depends on θ can be measured by the magnitude of δ, a quantity that is much easier to be determined than $\langle \varepsilon_\theta | \theta \rangle$. This relation is therefore very useful for turbulence and combustion modelers who may require the knowledge about the $\varepsilon_\theta - \theta$ dependence. Anselmet *et al.* [29] concluded from their boundary-layer and jet data that "the assumption of statistical independence between ε_θ and θ is sound in regions where θ fluctuations are almost symmetrical." Based on the present work, however, their conclusion is not accurate and should be modified to read "… where $p(\theta)$ is closely Gaussian." To support this point unambiguously, further comments are made here on the data of $p(\theta)$ and $q_1(\theta)$ reported in Jayesh and Warhaft [6] for the grid turbulence. In that case, for example, $p(\theta)$ is nearly symmetrical at the last measurement station. However, the corresponding $q_1(\theta)$ varies with θ in a V-shaped fashion (see Fig. 10 of Jayesh and Warhaft [6]), reflecting a strong $\varepsilon_\theta - \theta$ dependence (which is indeed quantified by $R_1 \approx 0.4$). This is obviously at odds with the conclusion of Anselmet *et al.* [29] but can be well explained in the context of the present work.

IV. CONCLUSIONS

The present study has examined the detailed statistical dependence of ε_θ on θ by directly measuring $\langle \varepsilon_\theta | \theta \rangle$ in two turbulent flows, i.e., a circular cylinder wake and a round jet. The correlation between this dependence and the departure of the PDF from Gaussian has been investigated.

It has been found that the departure of $p(\theta)$ from Gaussian is strongly related to the degree to which ε_θ depends on θ. This relationship can be quantified well by Eq. (6) or in more detail by

$$\frac{\langle \theta^2 \varepsilon_\theta \rangle}{\langle \theta^2 \rangle \langle \varepsilon_\theta \rangle} = C(|S_\theta| + |F_\theta - 3|) + 1 \qquad (7)$$

with $C \approx 0.36$. Based on our data obtained in the wake and jet flows and also those derived from previous measurements for grid turbulence and boundary layer, the empirical relation (7) is considered to be generic. According to Eq. (7), only when both $S_\theta = 0$ and $F_\theta = 3$, the equality $\langle \theta^2 \varepsilon_\theta \rangle = \langle \theta^2 \rangle \langle \varepsilon_\theta \rangle$ for the $\varepsilon_\theta - \theta$ independence is valid. Namely, the assumption of independence between ε_θ and θ, commonly used by combustion modelers, works when and only when the scalar PDF becomes Gaussian. This applies for both homogeneous and inhomogeneous turbulent flows.

[1] W. R. Hawthorne, D. S. Wedell, and H. C. Hottel, *Third Symposium (International) on Combustion, Flames and Explosion Phenomena* (The Combustion Institute, Pittsburgh, PA, 1949), pp. 266–288.
[2] S. B. Pope, Prog. Energy Combust. Sci. **11**, 119 (1985).
[3] S. B. Pope, *Twenty-Third Symposium (International) on Combustion* (The Combustion Institute, Pittsburgh, PA, 1990), pp. 591–612.
[4] E. E. O'Brien, *Turbulent Reacting Flows*, edited by P. A. Libby and F. A. Williams (Springer, Berlin, 1980), pp. 185–218.
[5] P. A. Libby and F. A. Williams (editors), *Turbulent Reacting Flows* (Academic, New York, 1994).
[6] Jayesh and Z. Warhaft, Phys. Fluids A **4**, 2292 (1992).
[7] Jayesh and Z. Warhaft, Phys. Rev. Lett. **67**, 3503 (1991).
[8] J. Mi, R. A. Antonia, and F. Anselmet, Phys. Fluids **7**, 1665 (1995).
[9] R. A. Antonia and J. Mi, J. Fluid Mech. **250**, 531 (1993).
[10] J. Mi, *The 1995 Australian Symposium on Combustion and the Fourth Australian Flame Days*, Adelaide, Nov. 9–10, 1995.
[11] V. Eswaran and S. B. Pope, Phys. Fluids **31**, 506 (1988).
[12] O. Metaisand and M. Lesieur, J. Fluid Mech. **280**, 157 (1992).
[13] F. A. Jaberi, R. S. Miller, C. K. Madnia, and P. Givi, J. Fluid Mech. **313**, 241 (1996).
[14] G. K. Batchlor, *The Theory of Homogeneous Turbulence* (Cambridge University Press, Cambridge, 1953).
[15] F. Heslot, B. Castaing, and A. Libchaber, Phys. Rev. A **36**, 5870 (1987).
[16] B. Castating, G. Gunaratne, F. Heslot, L. Kadanoff, A. Libchaber, S. Thomae, X.-Z. Wu, S. Zaleski, and G. Zanetti, J. Fluid Mech. **204**, 1 (1989).
[17] M. Sano, X. Z. Wu, and A. Libchaber, Phys. Rev. A **40**, 6421 (1989).
[18] Y. G. Sinai and V. Yakhot, Phys. Rev. Lett. **63**, 1962 (1989).
[19] V. Yakhot, Phys. Rev. Lett. **63**, 1965 (1989).
[20] A. Y. Klimenko, Phys. Fluids **7**, 446 (1994).
[21] V. R. Kuznetsov and V. A. Sabelnikov, *Turbulence and Combustion* (Hemisphere, New York, 1989).
[22] S. B. Pope and E. S. C. Ching, Phys. Fluids A **5**, 1529 (1993).
[23] E. S. C. Ching, Phys. Rev. Lett. **70**, 283 (1993).
[24] J. Mi, D. S. Nobes, and G. J. Nathan, J. Fluid Mech. **432**, 91 (2001).
[25] J. Mi and G. J. Nathan, Exp. Fluids **34**(6) 687 (2003).
[26] R. A. Antonia, B. R. Satyaprakash, and A. K. M. F. Hussain, Phys. Fluids **23**, 695 (1980).
[27] W. K. George and H. J. Hussein, J. Fluid Mech. **233**, 1 (1991).
[28] C. Tong and Z. Warhaft, J. Fluid Mech. **292**, 1 (1995).
[29] F. Anselmet, H. Djeridi, and L. Fulachier, J. Fluid Mech. **280**, 173 (1994).
[30] C. Tong and Z. Warhaft, Phys. Fluids **6**, 2165 (1994).
[31] E. E. O'Brien and T. L. Jiang, Phys. Fluids A **3**, 3121 (1991).